Modeling of bimorph piezoelectric cantilever beam for voltage generation

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ABSTRACT

Piezoelectric materials (PZT) have shown the ability to convert mechanical forces into an electric field in response to the application of mechanical stresses or vice versa. This property of the materials has found extensive applications in a vast array of areas including sensors and actuators. The study presented in this paper targets the modeling of PZT bender for voltage and power generation by transforming ambient vibrations into electrical energy. This device can potentially replace the battery that supplies the power in a micro watt range necessary for operating sensors and data transmission. One of advantages is the maintenance free over a long time span. This paper focuses on the analytical approach based on Euler-Bernoulli beam theory and Timoshenko beam equations for the voltage and power generation, which is then compared with two previously described models in literature; Electrical equivalent circuit and Energy method. The three models are then implemented in Matlab/Simulink/Simpower environment and simulated with an AC/DC power conversion circuit. The results of the simulation and the experiment have been compared and discussed.

Keywords: PZT bimorph, piezoelectric generator, PZT modeling

1. INTRODUCTION

The idea of building portable electronic devices or wireless sensors that do not rely on power supplies with a limited lifespan has intrigued researchers and instigated a sharp increase in research area of power harvesting. One method of power harvesting is the use of piezoelectric materials (PZT), which form transducers that are able to interchange between electrical energy and mechanical strain or force. Therefore, these materials have been employed as media to transform ambient motion (usually vibration) into electrical energy that can be stored and used to power electronic devices. Integration of the power harvesting device into sensor suites can allow for free maintenances in comparison with the use of battery that commonly requires a periodic replacement. In addition, there are many applications where sensors are physically embedded in an environment and no more accessible for a replacement. Moreover, the physical properties to be measured in the environment vary either relatively slow or do not need continuously to be processed for a high hierarchical system. Consequently, these sensor systems can be effectively operated by intermittent transmission of data gathered and the associated power consumption can be reduced.

Previous publications and patents indicate extensive application potential of a PZT (Lead Zirconate Titanate) power harvesting device as a prospective replacement for the batteries currently employed. The electrical and mechanical behaviour of the PZT power harvesting device has been studied by employing various approaches.

Umeda, et al [1] were among the pioneers to study the PZT generator and proposed an electrical equivalent model being converted from mechanical lumped models of a mass, a spring and a damper that describe a transformation of the mechanical impact energy into electrical energy in the PZT material. Ramsay and Clark [2] considered effects of transverse force on the PZT generator in addition to the force applied in the poling direction. Kasyap et al [3] formulated a lumped element model that represents the dynamic behavior of the PZT device in multiple energy domains and replace with electric circuit components. The model has been experimentally verified by using a one dimensional beam structure. Gonzalez et al [4] analyzed the prospect of the PZT based energy conversion, and suggested several issues to raise the electrical output power of the existing prototypes to the level being theoretically obtained.

Smits and Chio [5] studied the electromechanical characteristics of a heterogeneous piezoelectric bender subject to various electrical and mechanical boundary conditions based on internal energy conservation. However, the model used does not provide any formulation for the voltage generation. Other authors such as Huang et al. [6] and DeVoe et al. [7]

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did the displacement and tip-deflection analysis along the beam and made a comparison with the experimental results. However, both proposals were limited to the actuator mode.

Hwang and Park [8] introduced a new model that is extracted form the calculation of the FEM (Finite Element Method) and calculated the static responses of a piezoelectric bimorph beam in a piezoelectric plate element. However, no comparison has been made with experiments. Williams et al. [9] analyzed a PZT structure by using a single degree of freedom mechanical model. However, the model did not extend to a bimorph multilayer structure. Roundy et al. [10-12] presented a slightly different approach based on the electrical equivalent circuit to describe the PZT bender, which leads to fair matches with the experimental results. However, the analysis only considered a low-g (1-10 m/s2) vibration condition and lacks mechanical dynamics of the structure. Another authors, Lu et al. [13], improved the electrical model by adding an electro-mechanical coupling that represents a dynamic behavior of the beam vibrating under a single degree of freedom. Eggborn [14] developed the analytical models to predict the power harvesting from a cantilever beam and a plate using Bernoulli-beam theory and made a comparison with the experimental result. However the structure used the study doesn't have a proof mass attached at the end of the beam. Kim [15] analyzed the unimorph and bimorph diaphragm structure for the power generation using energy generation and piezoelectric constitutive equations. However, this study was limited to only diaphragm structures that were optimized through numerical analysis and FEM simulation at higher acceleration condition. Shen et al. [16] investigated the parameters influencing the output energy of piezoelectric bimorph cantilever beam with a proof mass, where the resonant frequency and robustness of a cantilever structure are considered for enhancing power conversion efficiency and implementing devices at high acceleration conditions.

The above studies have all had some success in modeling the PZT cantilever beam for voltage and power generation. However many issues such as extensive theoretical analysis of bimorph piezoelectric power generator based on cantilever beam structure with proof mass attached at the end have not been addressed fully and lacks modeling of power conversion circuitry. In this paper, special emphasis has been given to the analytical modeling of a bimorph PZT bender with a proof mass in the generator mode. The mathematical models developed are implemented in Matlab/Simulink with AC/DC power conversion circuitry. Models developed for this application are then compared with the experimental results to assess the accuracy of the various models.

2. MATHEMATICAL MODELS

Several different modeling approaches have been applied to study the dynamic characteristics of the structure. Most of works published have applied an electric equivalent circuit to represent the mechanical characteristics of the structure, which does not fully reflect actual dynamics of the structure. Euler-Bernoulli beam theory has also been previously studied for a unimorph structure but has been limited to modeling in the actuation mode. Thus, a new approach based on combination of Euler-Bernoulli beam theory and Timoshenko beam equation has been developed for the bimorph PZT bender taking into account material properties and coupled with the power conversion circuit. Fig 1 shows a schematic diagram of a PZT cantilever beam.

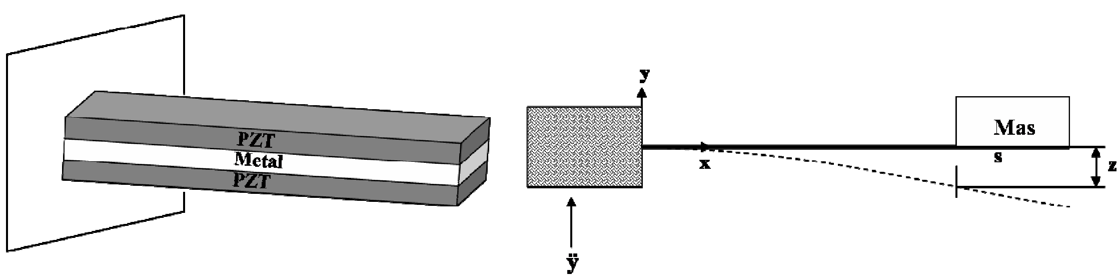

Fig. 1. A schematic diagram of a PZT cantilever beam.

The following section describes the development of three mathematical models aforementioned for the device. The first model is the based on combination of Euler-Bernoulli beam theory and Timoshenko beam equation. The second model is based on an electrical equivalent circuit and the third model is based on energy method.

2.1 Beam theory (Timoshenko and Euler-Bernoulli)

The static analysis of a piezoelectric cantilever sensor is typically performed by the use of calculations employed for deflection of a thermal bimorph proposed by Timoshenko [5-7]. The principle is based on the strain compatibility between three cantilever beams joined along the bending axis. Due to forces applied by one or all of the layers, the deflection of the three-layer structure is derived from a static equilibrium state. The structure considered is a piezoelectric heterogeneous bimorph, where two piezoelectric layers are bonded on both sides of a purely elastic layer, i.e., brass.

Fig. 1 shows a basic geometry of the three-layer multi-morph. A brass with a pure elasticity is sandwiched between the upper and lower layers of the PZT material. The modeling of this structure neglects shear effects and ignores residual stress-induced curvature. In addition, the beam thickness is much less than the piezoelectric-induced curvature, so the second order effects such as electrostriction can be ignored.

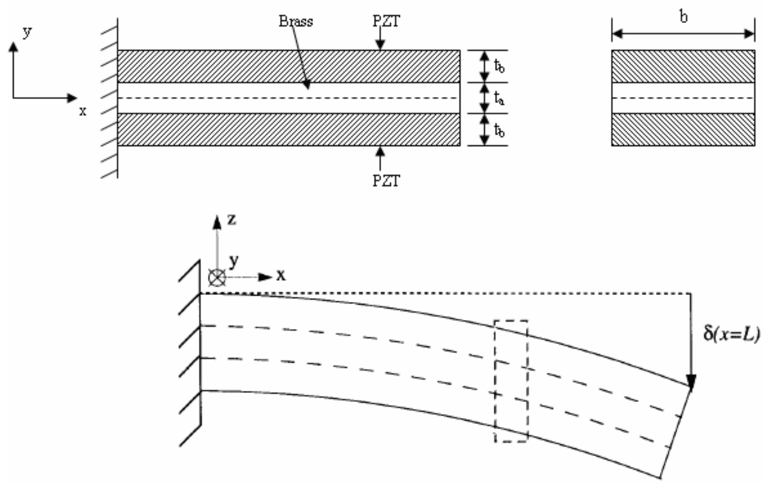

Fig. 1. Geometry of the beam.

Moreover, the radius of curvature for all the layers is assumed approximately to be the same to those of the structure, simply because of the assumption that the thickness is much less than the overall beam curvature.

The total strain at the surface of each layer is the sum of the strains caused by the piezoelectric effect, the axial force, and the bending. It is noted that the sign of the surface strain depends on the bending of either the top or bottom surface of the layer;

$$\varepsilon_i = \varepsilon_{\text{piezo}} + \varepsilon_{\text{axial}} + \varepsilon_{\text{bend}} = d_{31}E_i + \frac{F_i}{A_i Y_i} \pm \frac{t_i}{2r} \tag{1}$$

$\varepsilon_{\text{piezo}}$ in the linear constitutive equation above considers the transverse piezoelectric coupling coefficient d_{31} and the electric field across the thickness of the layer E_i for a piezoelectric material, t_1 and t_3 are the thickness of the PZT layer and t_2 is thickness of the center shim, A_i is the area of the corresponding layer and Y_i is the Young's modulus of elasticity. Hence the radius of curvature is given by the term

$$\frac{1}{r} = \frac{2d_{31}DA^{-1}C}{2 - DA^{-1}B} \tag{2}$$

Where

$$A = \begin{bmatrix} \frac{1}{A_1Y_1} & -\frac{1}{A_2Y_2} & 0 \\ 0 & \frac{1}{A_2Y_2} & -\frac{1}{A_3Y_3} \\ 1 & 1 & 1 \end{bmatrix} B = \begin{bmatrix} t_1 + t_2 \\ t_2 + t_3 \\ 0 \end{bmatrix} C = \begin{bmatrix} -E \\ E \\ 0 \end{bmatrix} \tag{3}$$

On the other hand, Euler-Bernoulli beam theory describes the relationship between the radius of curvature and the force applied, which is given by the following equation [14]

$$\rho A \frac{\partial^4 w(x,t)}{\partial t^4} + YI \frac{\partial^4 w(x,t)}{\partial x^4} = F(t) \tag{4}$$

Where ρ is the density, I is the moment of inertia and $F(t)$ is the applied force. A general solution for this equation is given by

$$w(x,t) = \sum q_i(t) X_i(x) \tag{5}$$

where the displacement and the vibration is expressed in the case of a cantilever beam as follows:

$$X_i(x) = Cosh(\beta_i x) - Cos(\beta_i x) - \frac{Sinh(\beta_i L) - Sin(\beta_i L)}{Cosh(\beta_i L) + Cos(\beta_i L)} \left(Sinh(\beta_i x) - Sin(\beta_i x) \right) \tag{6}$$

$$q_i(t) = \frac{1}{\omega_{di}} e^{-\zeta \omega_{ni} t} \int_0^t F_i(\tau) e^{-\zeta \omega_{ni} t} \sin\left(\omega_{di}(t - \tau)\right) d\tau \tag{7}$$

and

$$\beta_i^4 = \frac{\omega_{ni}^2}{C^2} \tag{8}$$

ω_n, is the natural frequency obtained by solving the transcendental equation;

$$Cosh(\beta_i L) Cos(\beta_i L) + 1 = 0 \tag{9}$$

The radius of curvature is given by the following equations:

$$r = \frac{1}{2w(L)} L^2, \text{ where } \frac{1}{r} = \dot{w}(x) \text{ and } w(x) = \frac{1}{2r} x^2 \tag{10}$$

Hence by substituting the radius of curvature term in the equation (6), the voltage produced for the PZT bender is given by:

$$V = \frac{2w(L)t_p}{L^2} \frac{2 - DA^{-1}B}{2d_{31}DA^{-1}} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}^{-1} \tag{11}$$

2.2 Electrical Equivalent Circuit

The study of the transient dynamic characteristics of a PZT bender utilizing electrical equivalent models has been performed in previous studies and the model has shown fair accuracy in various conditions of mechanical stress [10-13]. The electrical equivalent model has been studied and implemented in this research effort to compare the accuracy and validity of the experimental results and the analytical results from the models based in beam theory and energy conservation. Figure 3 shows an electric equivalent circuit model for a PZT beam [11], where a voltage source are connected in series with an inductor, a resistor and a capacitor that build a resonant circuit. The transformer represents the voltage adaptation while the capacitor indicates the inherent capacitance of the device.

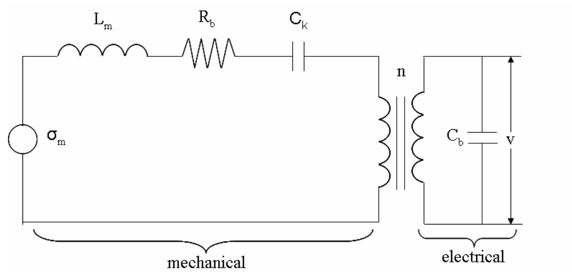

Fig. 3. Circuit representation of a PZT beam.

The circuit can be described by using Kirchhoff's voltage law:

$$\sigma_{in} = L_m \varepsilon + R_b \dot{\varepsilon} + \frac{\ddot{\varepsilon}}{C_k} + nV \tag{12}$$

$$i = C_{\mathrm{k}} \dot{V} \tag{13}$$

The equivalent circuits leads to the correlation between the strain *ε,* and voltage *V* [12]*;*

$$\ddot{\varepsilon} = \frac{-Y}{k_1 k_2 m}\varepsilon - \frac{b_{\mathrm{m}}}{k_1 m}\dot{\varepsilon} + \frac{Y}{k_1 k_2 m}\frac{d_{31}}{2t_{\mathrm{c}}}V + \frac{\ddot{y}}{k_2} \tag{14}$$

$$V = \frac{n_{\mathrm{p}} t_{\mathrm{c}} d_{31} Y}{\varepsilon}\dot{\varepsilon} \tag{15}$$

2.3 Conservation of energy

The principle is based on the fact that the total energy of the PZT bender stored is equal to the sum of the mechanical energy applied to the beam and the electric energy on the charges being applied by electric field [15, 17, 18]. When a mechanical stress applied, the energy stored in a PZT layer is the sum of the mechanical energy and the electric field induced energy. Thus, the energy stored in a PZT layer is expressed as follows;

$$U_{\mathrm{u}} = \frac{1}{2}\left(s_{11}^E \sigma_1 - d_{31} E_3\right)\sigma_1 = \frac{1}{2} s_{11}^E \sigma_1^2 \tag{16}$$

Where σ is the stress, s is the stiffness matrix. On the other hand, the energy in the metal layer can be expressed with a simple equation because of the lack of the electric field as follows;

$$U_{\mathrm{m}} = \frac{1}{2} s_{\mathrm{m}} \sigma_1^2 \tag{17}$$

The total energy of the beam is given as [15];

$$U_{\text{total}} = \int_0^L \int_0^W \left(\int_{\frac{t_1}{2}}^{\frac{t_2}{2}+t_1} \mathrm{d}U_{\text{u}}\mathrm{d}z + \int_{-\frac{t_2}{2}}^{\frac{t_2}{2}} \mathrm{d}U_{\text{m}}\mathrm{d}z + \int_{-\frac{t_2}{2}-t_3}^{-\frac{t_2}{2}} \mathrm{d}U_{\text{l}}\mathrm{d}z \right) \mathrm{d}y\mathrm{d}x \tag{18}$$

On the other hand, the electric field is given by $E = V/(2t_3)$. The total electrical energy is equal to a product of the charge and the voltage. Thus, the charge generated in the beam is obtained by a partial derivative of the total energy with respect to the voltage.

$$Q = \frac{\partial U_{\text{total}}}{\partial V} = -3\frac{d_{31}s_{\text{m}}(t_2+t_3)L^2}{X}F_o \tag{19}$$

The capacitance of the piezoelectric material is described as the relation between the voltage and charge on the piezoelectric material, hence the capacitance C_{free} of the beam can be found, where no load is applied [15].

$$C_{\text{free}} = \frac{\nu_{33}^T WL}{2t_3}\left(1 + \frac{\left(6s_{\text{m}}t_3(t_2+t_3)^2 - X\right)}{X}K_{31}{}^2\right) \tag{20}$$

Where K_{31} is the coupling coefficient. Thus, the voltage generated is found as a function of the applied force;

$$V = \frac{Q}{C_{\text{free}}} = -\frac{6d_{31}s_{\text{m}}t_3(t_2+t_2)L}{\nu_{33}^T WX\left(1+\left(\frac{6s_{\text{m}}t_3(t_2+t_3)^2}{X}-1\right)K_{31}{}^2\right)}F_o \tag{21}$$

The schematic structure of a sensor is shown in Fig. 4, where a mass (M_{end}) is attached to the free end of the bimorph PZT cantilever beam that is fixed to a vibrating base. Both of piezoelectric bending composite beam and M_{end} are assumed to be rigid bodies and no elastic coupling. Then, the structure can be modeled with a single degree of freedom (SDOF) system, which solely consists of a proof mass M, a spring with stiffness K, a damper with damping coefficient C and a vibrating base. The resulting equivalent model is shown in Fig. 4. Hence, *y(t)* is the motion of the vibrating base, and *z(t)* is the relative motion between the vibrating base and the proof mass M that is assumed to be a point mass with equivalent vertical force at the free end of the sensor. Thus, the mass can be expressed by a following equation [19];

$$M = \frac{33}{140}M_{\text{beam}} + M_{\text{end}} \tag{22}$$

Where M_{beam} is the mass of the beam and M_{end} is the end mass.

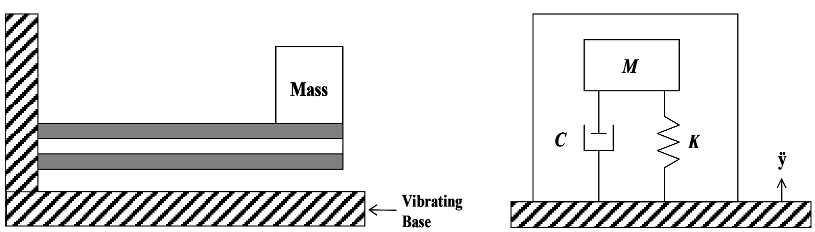

Fig. 4. Sensor structure and equivalent SDOF model.

According to the Newton's second law, the mechanical model is derived as follows:

$$M\ddot{z} + C\dot{z} + Kz = -M\ddot{y} \tag{23}$$

A transfer function between the input acceleration and the output displacement can be obtained in the Laplace plane with initial conditions $z(0) = \dot{z} = 0$, where, $\omega_n = \sqrt{\frac{K}{M}}$, $\zeta = damping\_ratio$.

$$\left|\frac{Z(s)}{\ddot{Y}(s)}\right| = \frac{1}{s^2 + (2\zeta\omega_n)s + \omega_n^2} \tag{24}$$

So, the response of the force F_o at the beam is obtained after $Z(t)$ and $\ddot{Z}(t)$ is solved from the equation to get

$$F_o(t) = M_{\text{end}} \times \ddot{Z}(t) \tag{25}$$

All of models described above are solved by using Matlab/Simulink. Simulation results are compared with the experimental results in the following chapters.

$$V = -\frac{6d_{31}s_{\text{m}}t_3(t_2+t_3)L}{v_{33}^T WX\left(1+\left(\frac{6s_{\text{m}}t_3(t_2+t_3)^2}{X}-1\right)K_{31}{}^2\right)} M_{\text{end}} \times \ddot{Z}(t) \tag{26}$$

3. ELECTRICAL CIRCUIT

The analytical models and the analysis is based on a simple resistive load is not a very realistic approximation of the actual electrical load. In reality, the electrical system would look something like the circuit shown in Figure 5. The equivalent mechanical side of the circuit is exactly the same as shown in Fig3. The development of a model for this case is useful in that it represents a more realistic operating condition.

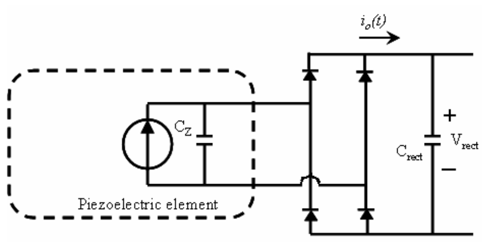

Fig. 5. A simplified circuit representation.

The major components involved in this circuit are; AC/DC rectifier and a filter capacitor. The AC/DC rectifier converts the AC signal from the piezo-source into DC current and the filter capacitor smoothes electrical flow.

4. EXPERIMENTS

The bender was composed of a brass center shim sandwiched by two layer made of a sheet of PZT-5A. The thickness of the brass plate and the PZT is 0.134mm and 0.132mm, respectively and the attached mass made from Tungsten. In order to investigate parameters of prototype structure, a test stand is built to excite the bender with a predetermined resonant frequency. The system described here is designed to utilize the z-axis vibration as the only vibration source for the device. The cantilever is excited by a shaker connected to a function generator *via* an amplifier. For a characterization of the fabricated cantilever device, the voltage generated was evaluated by connecting a resistor. Fig. 6 illustrate the schematic of experimental setup and a photo for a real setup.

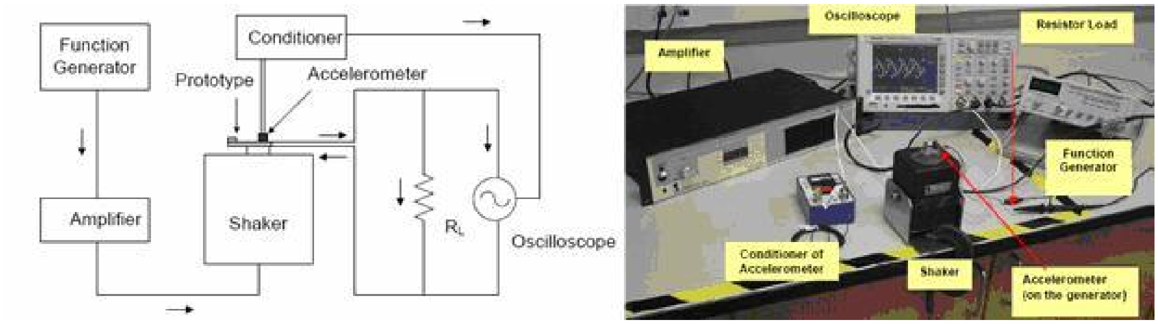

Fig. 6. Schematic and photo of experimental setup with a resistance load.

5. RESULTS AND DISCUSSION

The accuracy of the model was compared against experimental results to demonstrate the ability of the model to accurately predict the amount of power produced by the PZT generator when subjected to transverse vibration. To ensure that the model and experimental tests were subjected to the same excitation force an accelerometer was used to calculate the amplitude of the sinusoidal force applied to the beam. The beam was excited by a sinusoidal input and the steady state power output was measured across several different resistors. In order to examine the models, the power generated by piezoelectric prototypes were compared and evaluated. Three cases have been studied with an open circuit, a resistive load without and with a rectifier with a capacitor.

5.1 Open circuit

The output voltage waveforms obtained from the simulation and the experiments performed on the PZT bender are compared in Figure 7. The experimental results show 11.49V, while the models do 10.47V, 11.649V and 10.254V. Secondly, the phase displacements vary in a range of more than 90^0. The predicted response shown in the figure shows a transient response for a small period of time while the experimental results do not because they were recorded at steady state vibration. The results indicate that the models provide a very accurate measurement of the open circuit voltage generated.

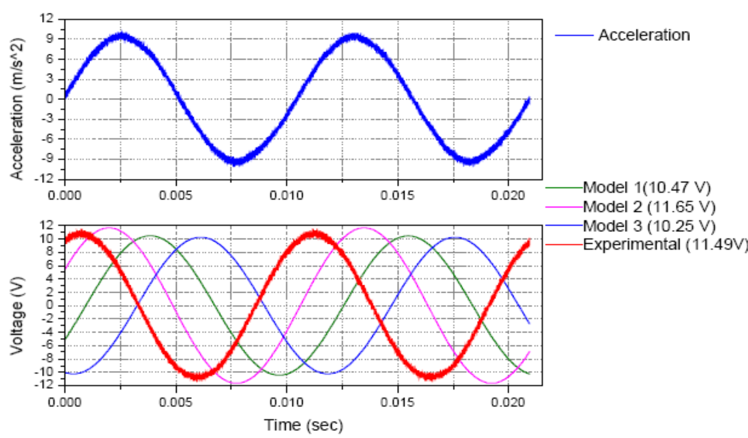

Fig. 7. Comparison of amplitude of the open circuit AC voltage for three models with experimental results.

5.2 Resistive load

Figure 8 and Figure 9 shows the output waveform of the PZT power generator measured and simulated, where a 4kΩ resistor is connected as a load. The peak voltage measured amounts to 0.58V, while the simulated are 0.521V, 0.713V and 0.553V for the three models, respectively.

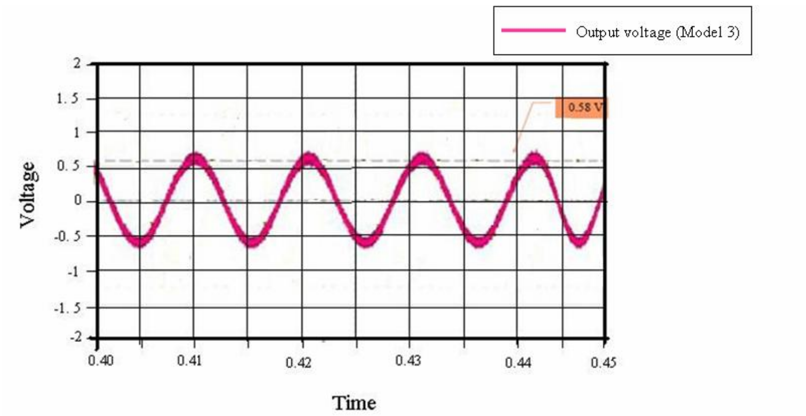

Fig. 8. Experimental results for the output voltage with a 4 kΩ resistive load.

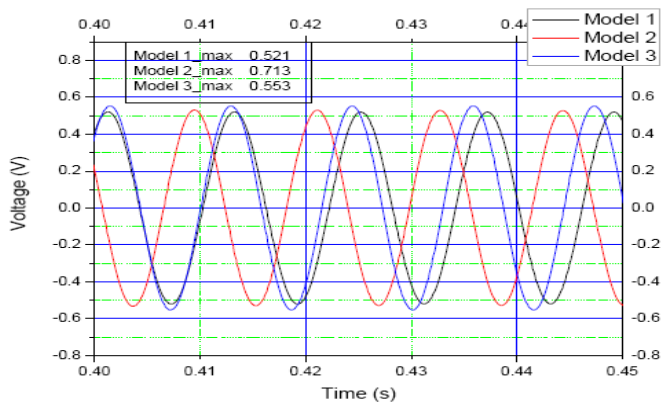

Fig. 9. Simulation results for the output voltage with a 4kΩ resistive load.

The experiments undertaken demonstrates that the system designed can supply a maximum power of 250 μW at 110 kΩ resistive load when the PZT bender is excited with a vibration with an amplitude of 9.8 m/s^2 (1-g) at 97.6 Hz.

5.3 Resistive load with rectifier

Multi run simulations have been carried out to compare both results. The model of the PZT beam is integrated into SIMPOWER by using a controlled voltage source. Figure 10 shows an integrated model with a PZT bender, a bridge rectifier with a capacitor and a resistor that has been implemented in Matlab/Simulink/Simpower.

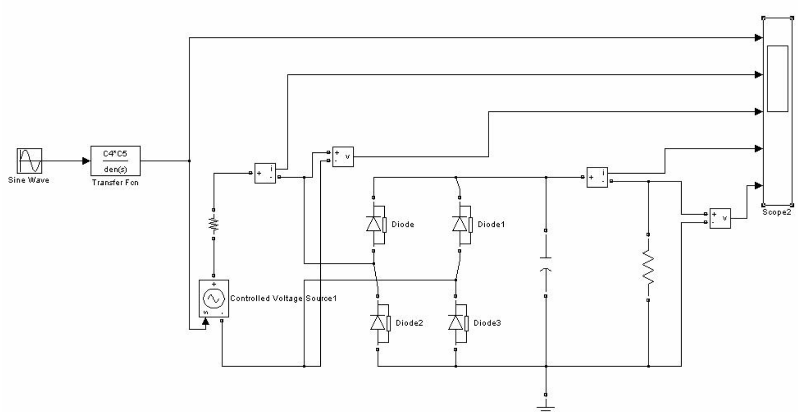

Fig. 10. Simulation with Matlab/Simulink/Simpower.

Figure 11 shows simulated results for AC voltage and AC current, and DC voltage for the transverse vibration of amplitude 1-g at the resonance frequency of the PZT bender. It is noted that the AC voltage clamps whenever the current starts to flow. It can also be interpreted that a voltage drop at the internal resistance drastically increases as soon as a current flows. It is noted that the current charging the DC capacitor is not sinusoidal and the influence of the current has been worsened at a resistive load with a rectifier compared to the previous case.

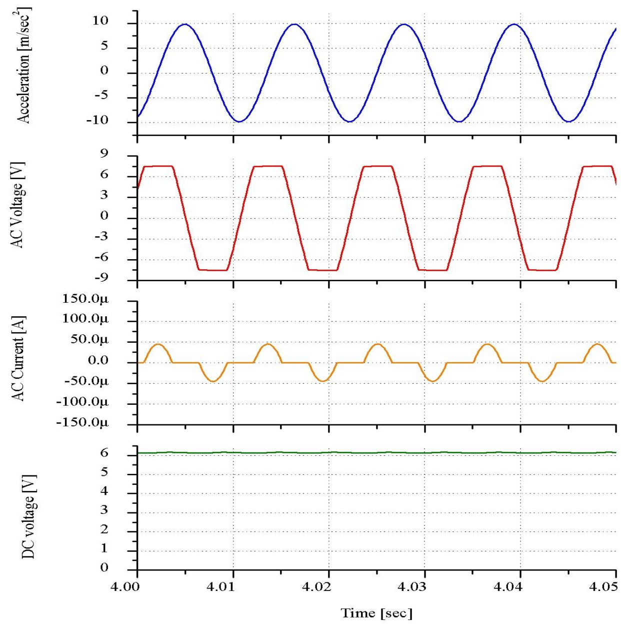

Fig. 11. AC voltage and current, and DC voltage at a 400kΩ resistive load with a rectifier.

6. CONCLUSION

One method of performing power harvesting is to use PZT materials that can convert the ambient vibration energy surrounding them into electrical energy. This electrical energy can then be used to power other devices or stored for later use. The need for power harvesting devices is caused by the use batteries as power supplies for the wireless electronics. As the battery has a finite lifespan, once extinguished of its energy, the sensor must be recovered and the battery replaced for the continued operation of the sensor. This practice of obtaining sensors solely to replace the battery can become and expensive task. Therefore, methods of harvesting the energy around these sensors must be implemented to expand the life of the battery or ideally provide an endless supply of energy to the sensor for its lifetime. A PZT bender with a bimorph structure is designed for a power generator. The 31 mode operation for the material is chosen because of the higher strain and lower resonant frequencies compared to those in the 33 mode operation. The work presented has been focused on modeling of the PZT materials in a cantilever beam structure and analyses of the device in conjunction with a power conversion circuit.

We have developed a model to predict the amount of power capable of being generated through the vibration of a cantilever beam with attached PZT elements. The derivation of the model has been provided with boundary conditions. The model was verified using experimental results and proved to be very accurate independent of load resistance. In addition, the verification of the model was performed on a bimorph PZT bender, indicating that the model is robust and can be applied to a variety of different mechanical conditions. The model developed provides a design tool for developing power harvesting systems by assisting in determining the size and extent of vibration needed to produce the desired level of power generation. The potential benefits of power harvesting and the advances in low power electronics and wireless sensors are making the future of this technology look very bright.

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